# The heating of a turbulent water jet discharged vertically into a steam environment

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(Received 15 April 1987 and in final form 9 November 1987)

Abstract—Available experimental results for the heating of turbulent water jets discharged downward into a steam environment are reviewed in terms of the Kutateladze theory for such a system. That theory defines an eddy diffusivity for heat that is proportional to the local jet Reynolds number,  $\varepsilon_{\rm H}/\nu = E(ur_1/\nu)$ , and the factor E is evaluated for the experimental results. The large range in the values of E so obtained remains essentially unexplainable and the design problem of specifying the heating of the water jet remains unresolved despite the very substantial experimental effort that has been devoted to this problem.

## INTRODUCTION

THE HEATING of a vertical jet of a single component, discharged downward into a region of its saturated vapor, has received considerable experimental attention in terms of the heating of a water jet. Kutateladze [1] produced an analysis in which the jet velocity was taken to be invariable with respect to radius, and in which turbulent transport was accounted for by an eddy diffusivity for heat proportional to the product of the local velocity and local jet radius,  $\varepsilon_{\rm H}/\nu = E(ur_1/\nu)$ . He indicated that the early results of Zakharov and Chernaya could be rationalized with  $E = 5 \times 10^{-4}$ , though this did not apply for the results of Zinger [2].

Many other results for the heating of water jets have been produced. Mills et al. [3] compared the Stanton numbers for a number of these and indicated that the substantial differences between them appeared to be accountable only in effects associated with differences in the nozzles from which the jets were produced. This paper reviews the same results, and includes others that have become recently available, using the Kutateladze theory to evaluate E for the various results. In it, the Kutateladze theory is reviewed, primarily because of some questions regarding its formulation, and as a convenience to the reader. Then the experimental results are indicated; for many of them the variety of the parameters on which the correlation of the Stanton number is given requires the evaluation of E for a specific jet Reynolds number representative of the range of experimental conditions. The values of E so obtained are diverse and, considering all of the experimental results to be of equal merit, they must be related to the nature of the nozzle producing the jet in some yet undefinable way. Appraisals made for the break-up length of the jet do not help in this matter.

#### ANALYSIS

For a steady, axisymmetrically uniform downward flow of constant density, the equations of continuity, motion and energy, with the pressure taken as constant in the former and dissipation neglected in the latter, are

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial vr}{\partial r} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(v+\varepsilon_{\rm M})\frac{\partial u}{\partial r}\right] + g \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\alpha + c_{\rm H})\frac{\partial T}{\partial r}\right].$$
 (3)

From equation (1)

$$rv = -\int_{0}^{r} \frac{\partial u}{\partial x} r \,\mathrm{d}r. \tag{4}$$

If u is assumed to be independent of radius then equation (4) gives

$$v = -\frac{r}{2}\frac{\mathrm{d}u}{\mathrm{d}x}.$$
 (5)

Integration of equation (2) using equation (1) gives the integral form of the momentum equation

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_{0}^{r_{1}} u^{2}r \,\mathrm{d}r - u^{2}r_{1} \frac{\mathrm{d}r_{1}}{\mathrm{d}x} + r_{1}u_{1}v_{1}$$
$$= r(v + \varepsilon_{\mathsf{M}}) \frac{\partial u}{\partial r}\Big|_{r_{1}} + \frac{g}{2}r_{1}^{2}. \quad (6)$$

Without any addition of mass at  $r = r_1$ , the exterior of the jet,  $ur_1^2 = u_0r_0^2$ , and equation (5) gives

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# NOMENCLATURE

$C_p$	specific heat capacity of the liquid
	$[J kg^{-1} K^{-1}]$

- d nozzle diameter [m or mm]
- *E* dimensionless constant in Kutateladze theory
- g gravitational acceleration [m s<sup>-2</sup>]
- h heat transfer coefficient  $[J m^{-2} h^{-1} K^{-1}]$
- $h_{\rm fg}$  latent heat of evaporation [J kg<sup>-1</sup>]
- k molecular thermal conductivity  $[J m^{-1} s^{-1} K^{-1}]$
- 1 distance that the liquid jet travels through
  the steam space [m or mm]
- $l_{\rm N}$  nozzle length [m or mm]
- $m_c$  mass flow rate of liquid condensate added to the jet [kg s<sup>-1</sup>]
- $m_0$  initial mass flow rate at the nozzle exit  $[kg s^{-1}]$
- *p* system pressure [MPa]
- r distance from the axisymmetric axis
   [m or mm]
- *Re* nozzle Reynolds number,  $(u_0 d/v)$
- St Stanton number,  $h/(\rho c_p u_0)$
- T temperature of the jet;  $T_0$ , initial;  $T_s$ , saturation;  $T_m$ , mixed mean [°C]
- $T_{\rm s}$ , saturation,  $T_{\rm m}$ , inset inear [ C]  $T_{\rm R}$  temperature ratio,  $(T_{\rm s} - T_{\rm 0})/(T_{\rm s} - T_{\rm m})$
- u velocity of the jet [m s<sup>-1</sup>]
- v radial velocity of the jet [m s<sup>-1</sup>]

*We'* Weber number for the vapor,  $(\rho_v du_0^2/\sigma)$ 

- WeWeber number for the liquid,  $(\rho_L du_0^2/\sigma)$ xvertical distance from the end of the<br/>nozzle [m or mm]
- Z Ohnessorge number,  $\mu/\sqrt{(\rho d\sigma)}$ .

# Greek symbols

- $\alpha$  molecular diffusivity  $[m^2 s^{-1}]$
- $\varepsilon_{\rm H}$  eddy diffusivity of heat  $[m^2 s^{-1}]$
- $\varepsilon_{M}$  diffusivity of momentum [m<sup>2</sup> s<sup>-1</sup>]
- $\mu$  molecular viscosity [N s m<sup>-2</sup>]
- v kinematic viscosity  $[m^2 s^{-1}]$
- $\rho$  liquid density [kg m<sup>-3</sup>]
- $\sigma$  surface tension [N m<sup>-1</sup>]
- $\tau$  shear stress [N m<sup>-2</sup>].

#### Subscripts

- 0 initial value
- 1 at the outer edge of the jet
- B location at which the jet disintegrates into drops.

#### Superscripts

- average property
  - dimensionless quantity, i.e.  $r^* = r/r_1$ ,  $x^* = x/r_0$ .

$$v = \frac{r}{r_1} u \frac{\mathrm{d}r_1}{\mathrm{d}x}.$$
 (7)

This makes the sum of the second and third terms on the left-hand side of equation (6) zero. With no mass addition at the jet surface the shear is zero there; then the first term on the right-hand side of equation (6) is zero; then equation (6) gives

$$u = u_0 \left( 1 + \frac{2gx}{u_0^2} \right)^{1/2}$$
(8)

$$r_1 = r_0 \left( 1 + \frac{2gx}{u_0^2} \right)^{-1/4}.$$
 (9)

Transforming (x, r) to  $(x^*, r^*)$  in equation (3), with  $x^* = x/r_0$  and  $r^* = r/r_1$  gives

$$\frac{u}{r_0}\frac{\partial T}{\partial x^*} - \frac{ur}{r_1^2}\frac{dr_1}{dx}\frac{\partial T}{\partial r^*} + \frac{v}{r_1}\frac{\partial T}{\partial r^*} = \frac{1}{r_1^2 r^*}\frac{\partial}{\partial r^*} \left[ (\alpha + \varepsilon_{\rm H})r^*\frac{\partial T}{\partial r^*} \right].$$
(10)

For u invariable with r, equation (7) applies, making the sum of the second and third terms on the left-hand side of equation (10) zero to give

$$\frac{\mu}{r_0}\frac{\partial T}{\partial x^*} = \frac{1}{r_1^2 r^*}\frac{\partial}{\partial r^*} \left[r^*(\alpha + \varepsilon_{\rm H})\frac{\partial T}{\partial r^*}\right].$$
 (11)

This is the form used by Kutateladze [1]. Part of the foregoing development was included because of questions raised, as by Dement'yeva and Makarov [4], as to possible omission of the second and third terms of equation (10).

Kutateladze assumed the eddy diffusivity for heat,  $\varepsilon_{\rm H}$ , to be invariable with the radius, so that equation (11) could be stated as

$$\frac{ur_1^2}{r_0(\alpha+\varepsilon_{\rm H})}\frac{\partial T}{\partial x^*} = \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T}{\partial r^*}\right)$$
(12)

with

$$d\xi = \frac{r_0(\alpha + \varepsilon_H)}{ur_1^2} dx^*.$$
 (13)

Equation (12) becomes

$$\frac{\partial T}{\partial \xi} = \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial T}{\partial r^*} \right). \tag{14}$$

For  $T = T_0$ ,  $0 < r^* < 1$  for  $\xi = 0$  and  $T = T_s$  at  $r^* = 1$ , all  $\xi$ , the solution of equation (14) is

$$\frac{T-T_s}{T_0-T_s} = 2\sum_{n=1}^{\infty} \frac{J_0(\beta_n r^*)}{\beta_n J_1(\beta_n)} \exp\left(-\beta_n^2 \xi\right)$$
$$J_0(\beta_n) = 0. \tag{15}$$

For u invariable with r, the mean temperature  $T_m$  is

$$T_{\rm m} = \frac{2}{r_1^2} \int_0^{r_1} Tr \,\mathrm{d}r$$

and

$$\frac{T_{\rm m}-T_{\rm s}}{T_{\rm 0}-T_{\rm s}} = \sum_{n=1}^{\infty} \frac{4}{\beta_n^2} \exp{(-\beta_n^2 \xi)}.$$
 (16)

For the evaluation of equation (16), Isachenko *et al.* [5] give an approximation for the sum in equation (16); alternatively Hasson *et al.* [6] give an approximation for small values of  $\xi$ . The former was used for equation (16); the result agrees with ref. [6]. For  $[(T_s - T_0)/(T_s - T_m)] > 2.7$ , the first eigenvalue is sufficient in equation (16), and

$$\log \frac{T_{\rm s} - T_{\rm o}}{T_{\rm s} - T_{\rm m}} = 0.37 + 5.78\xi. \tag{17}$$

Kutateladze assumed  $\varepsilon_{\rm H} = E(ur_1)$ , then the integration of equation (13) gives

$$\zeta = \frac{\alpha}{u_0 r_0} x^* + \int E\left(\frac{r_0}{r_1}\right) \mathrm{d}x^*$$

if E is a constant then

$$\xi = \frac{\alpha}{u_0 r_0} x^* + EF x^* \tag{18}$$

where

$$F = \frac{1}{x^*} \int \frac{r_0}{r_1} dx^* = \frac{4}{5} \frac{u_0^2}{2gx} \left[ \left( 1 + \frac{2gx}{u_0^2} \right)^{5/4} - 1 \right]$$

*F* is the average value of  $r_0/r_1$  over the length of the jet, as  $2gx/u_0^2 \rightarrow 0$ ,  $F \rightarrow 1$ .

If E depends on  $x^*$  then the last term in the expression for  $\xi$  can be written as

$$\bar{E}Fx^* = \int_0^{x/r_0} E \frac{r_0}{r_1} \mathrm{d}x^*.$$

This is the kind of average E that is deduced from a specified value of  $\xi$  when equation (18) is used. If a heat transfer coefficient is defined as

$$h = \frac{qr_1/r_0}{T_s - T_m}$$

then a heat balance between x = 0 and x, together with  $ur_1^2 = u_0 r_0^2$ , gives the Stanton number as

$$\frac{\bar{h}}{\rho c_{p} \mu_{0}} = \frac{r_{0}}{2x} \log \frac{T_{s} - T_{0}}{T_{s} - T_{m}}; \quad \bar{h} = \frac{1}{x} \int_{0}^{x} h \, dx. \quad (19)$$

It is convenient to define  $T_{\rm R} = (T_{\rm s} - T_{\rm 0})/(T_{\rm s} - T_{\rm m})$  and this is used mostly hereafter.

As an alternative, equations (1)-(3) can be solved

numerically, and the mass addition to the fluid can then be included. With mass addition

$$-\rho v h_{\rm fg} = (k + \rho c_p \varepsilon_{\rm H}) \frac{\partial T}{\partial r} \quad \text{at } r = r_{\rm i} \qquad (20)$$

and

$$-\rho uv = \tau = \rho(v + \varepsilon_{\rm M}) \frac{\partial u}{\partial r} \quad \text{at } r = r_1.$$
 (21)

This was done for some conditions. A turbulence model is required. The only one used was  $\varepsilon_{\rm M} = \varepsilon_{\rm H}$  and  $\varepsilon_{\rm H} = E(ur_1)$  as assumed for equation (18).

With the ratio  $(T_s - T_0)/(T_s - T_m)$  given by analytical, numerical, or experimental results, there can be specified the total condensation in the length x

$$m' = -(\rho v_1) 2\pi r_1; \quad m_c = -2\pi \rho \int_0^x r_1 v_1 \, \mathrm{d}x.$$

An energy balance for constant liquid specific heat gives

$$(m_0 + m_c)(T_m - T_s) = m_0(T_0 - T_s) + m_c \frac{h_{fg}}{c_p}$$
$$\frac{m_c}{m_0} = \frac{T_R - 1}{1 + KT_R}; \quad K = \frac{h_{fg}}{c_p(T_s - T_0)}.$$
(22)

The experimental results give  $\log T_{\rm R}$ , or the Stanton number, for a given value of x/d. From this, equation (16) determines  $\xi$  and equation (18) produces a value of E from  $\xi$  and the values of  $r_0$  and  $u_0$ . For a true nozzle with a discharge coefficient of unity the initial radius is the nozzle exit radius and  $u_0$  is given from the measured mass flow rate,  $u_0 = m_0 / \rho \pi r_0^2$ . For turbulent flow through a nozzle which is a relatively long tube of radius  $r_0$ ,  $m_0 = \rho \pi r_0^2 \bar{u}$ . Then with  $u_0 = \bar{u}$ , there should be, in equation (8) the factor, f, on  $u_0$  which makes  $f^2 u_0^2/2$  the initial kinetic energy of the flow; the factor is close enough to unity to make its effect negligible. In the other limit, for a nozzle that is really a sharp edged orifice with  $c_D = 0.60$ ,  $m_0 = 0.60 \rho \pi r_0^2 u_t$ , where  $u_t$  is the velocity as calculated for the minimum flow cross section of radius r.. Now  $u_{\rm t}$  and  $r_{\rm t}$  are the 'zero' values to be used in equation (18). Since  $m_0 = \rho \pi r_t^2 u_t$ , then  $(r_t/r_0)^2 = 0.60$ . Defining a  $u_0$  by  $m_0 = \rho \pi r_0^2 u_0$  then  $(u_t/u_0) = 1.67$ . With this the quantities in equation (18) become

$$\frac{x}{r_{\rm t}} = 1.29 \frac{x}{r_{\rm 0}}; \quad \frac{2gx}{u_{\rm t}^2} = \frac{1}{(1.67)^2} \frac{2gx}{u_{\rm 0}^2}$$

and

$$\frac{\alpha}{u_t r_t} \frac{x}{r_t} = \frac{\alpha}{u_0 r_0} \frac{x}{r_0}.$$

If E is the value given by  $EFx^*$  (as from equation (18)) using the above and  $E_0$  is the value using  $u_0$  and  $r_0$  as the initial values then

$$0.96 > (E/E_0) > 0.83$$
 for  $21 > (2gx/u_0^2) > 1.5$   
 $0.83 > (E/E_0) > 0.77$  for  $1.5 > (2gx/u_0^2) > 0$ .

Even in this extreme case the ratio is not very significant compared to the uncertainty that might exist in E.

The uncertainty in E can be related to the uncertainty in the data,  $\log T_{\rm R}$ . For fixed  $u_0$ ,  $r_0$  and x, equation (18) gives

$$dE = \frac{d\xi}{Fx^*} = \frac{d(\log T_R)}{Fx^*} \frac{d\xi}{d(\log T_R)}$$

and  $d(\log T_R)/d\xi$  can be evaluated from equation (16); call this value s. Then

$$\frac{\mathrm{d}E}{E} = \frac{1}{EFx^*} \left( \frac{\mathrm{d}\log T_{\mathrm{R}}}{\log T_{\mathrm{R}}} \right) \left( \frac{\log T_{\mathrm{R}}}{s} \right)$$
$$\simeq \frac{1}{EFx^*} \left( \frac{\mathrm{d}\log T_{\mathrm{R}}}{\log T_{\mathrm{R}}} \right) \frac{(\log T_{\mathrm{R}})^n}{5.78} \quad (23)$$

where

$$n = 1.57$$
 for  $0.15 < \log T_{\rm R} < 1$   
 $n = 1$  for  $\log T_{\rm R} > 1$ 

as from equation (17).

In reference to the data, it is noted from equation (19) that

$$\frac{\mathrm{d}(\log T_{\mathrm{R}})}{\log T_{\mathrm{R}}} = \frac{\mathrm{d}(\bar{h}/\rho c_{p} u_{0})}{(\bar{h}/\rho c_{p} u_{0})}$$

Finally, because experimental results are often specified in terms of the Stanton number, it is noted that the analytical solution gives, from equations (19) and (16)

$$\frac{\bar{h}}{\rho c_{p} u_{0}} = \frac{r_{0}}{2x} \log\left(\frac{T_{s} - T_{0}}{T_{s} - T_{m}}\right) = \frac{1}{2x^{*}} f_{1}(\zeta)$$

and from equation (18)

$$\frac{\bar{h}}{\rho c_p u_0} = \frac{1}{2x^*} f_1\left(\frac{\alpha}{u_0 r_0} x^* + EFx^*\right)$$

where

$$F = f_2\left(\frac{gd}{u_0^2}x^*\right)$$

thus

$$\frac{\overline{h}}{\rho c_p u_0} = f_3\left(\frac{x}{r_0}, \frac{\alpha}{u_0 r_0}, \frac{gd}{u_0^2}, E\right)$$

The relations

$$f_1(\xi), f_2\left(\frac{gd}{u_0^2}\frac{x}{r_0}\right)$$

are complicated; there is no reason to expect a power law dependence between the variables in  $f_3$ . In it E is, at least, probably a function of the initial Reynolds number, and if E depends on x (E in  $f_3$  is then  $\overline{E}$ ) it may, as noted later, depend on the relative condensation,  $(m_c/m_0)$ , so that equation (22) adds the variable K, and  $E = f[(u_0r_0/\nu), K]$ . Also

$$\frac{gd}{u_0^2} = \left(\frac{v}{u_0d}\right)^2 \left(\frac{gd'}{v^2}\right)$$

Then

$$\frac{\bar{h}}{\rho c_{\rho} u_0} = f_3 \left( \frac{x}{r_0}, \frac{\alpha}{\nu}, \frac{gd}{u_0^2}, \frac{\nu}{u_0 r_0}, K \right)$$

or

$$f_3\left(\frac{x}{r_0}, \frac{\alpha}{\nu}, \frac{\nu}{u_0r_0}, \frac{gd^3}{\nu^2}, K\right).$$
(24)

## LENGTH OF THE CONTINUOUS JET

The foregoing analysis applies to the continuous region of the jet, between the nozzle and the location at which the jet breaks up into drops, and the specification of the length of the continuous region is still uncertain. There are specifications for both laminar and turbulent flow and mention of both is made here though the present consideration is limited to the latter.

For an initially laminar jet flow, it is fairly well established that for  $\sqrt{We} < 3$ ,  $We = \rho du_0^2/\sigma$ , drops form at the nozzle, and for  $3 < \sqrt{We} < (\sqrt{We})_2$  drops are formed from axisymmetrical waves at a location

$$\left(\frac{l}{d}\right) = A\sqrt{We}.$$
 (25)

(Here, and hereafter, l is used for the distance from the nozzle for which x was so far used. This change is made for correspondence with the notation used in most of the experimental results.)

For  $\sqrt{We} > (\sqrt{We})_2$  there is a region for which  $(l/d)_B$  is almost constant, and then  $(l/d)_B$  decreases toward an asymptotic value that is one-half or less than the maximum value of l/d; neither this asymptotic value or the  $\sqrt{We}$  at which it occurs are well defined. There are old results of Tyler, given by Bogy [7], obtained with very small diameter nozzles with relatively high initial velocities, for various fluids, for the narrow range 914 < Re < 1410, that indicate  $A \simeq 12$  for various values of the Ohnessorge number Z,  $Z = \mu/\sqrt{\rho d\sigma}$ , different mostly because of the properties of the various fluids. For most of the results

$$(\sqrt{We})_2 = 3.1 + 790Z.$$
 (26)

Iciek [8] indicates results for various fluids issuing from short cylindrical nozzles with length,  $l_N$ , giving  $(l_N/d) \simeq 1$  and also with such nozzles including a conical inlet of equal length. For those flows considered to be laminar, which for water gave *Re* as high as about 4000, the factor *A* was indicated to be

$$A = (8 - 2.5 \log Z)(1 + 3Z).$$
(27)

For Z = 0.0074, typical of the Tyler experiments with

water, for which A = 12, relation (27) gives the higher value of 21.

For turbulent flow Iciek [8] found the length of the continuous region was independent of nozzle length for cylindrical nozzles for  $(l_N/d) > 5$  and Re > 3000, and for such nozzles with rounded inlet edges for  $(l_N/d) > 15$  and Re > 4000. Then

$$\left(\frac{l}{d}\right)_{\rm B} = 11.5(\sqrt{We})^{0.62}.$$
 (28)

Relation (28) was established for values of  $\sqrt{We}$  up to 40. There was no indication of a maximum value of  $(l/d)_{\rm B}$ , at a  $(\sqrt{We})_2$ , as found for laminar flow.

## AVAILABLE RESULTS FOR COMPARISON

Many results are available for vertical water jets heated by downward discharge into a steam environment. These are given in terms of  $\log T_R$ , or equivalently, the Stanton number, as defined by equation (19). Those of Zakharov and Chernaya for jets produced by short cylindrical nozzles, are presented in ref. [1] and analyzed there to indicate that  $E = 5 \times 10^{-4}$  is relatively satisfactory. Other results, due to Zinger [2], are also shown in ref. [1].

Other experimental results are indicated in Table 1, which gives various correlations for the Stanton number, notable for the variety of the parameters the expressions include. For the results of Isachenko et al. [5] for a relatively long cylindrical nozzle,  $(l_N/d) = 46$ , the relation is that given in that reference. For Sklover and Rodivilin [9] this is also the case. This nozzle was presumably cylindrical with  $(l_N/d)$ small. The results were for a single jet and arrays of up to 46 jets, with  $T_{\rm m}$  determined by thermocouple traverses at various locations (l/d); apparently it was assumed that the velocity of the jet was invariable with radius, as in the theory, in the evaluation of  $T_{\rm m}$ . A range of pressures was examined, as was also a range of steam velocities, parallel to the jet flow. The effect of the latter was essentially negligible and its effect was not included in the correlation for the Stanton number.

In respect to the results of Isachenko *et al.* and Sklover and Rodivilin, it is to be noted that for them the Stanton number increases as  $u_0$  increases; this is true also for the Zinger results. For all of the other results, the Stanton number decreases as  $u_0$  increases.

The results of Iciek [10] are for cylindrical nozzles,  $1 < (l_N/d) < 8.7$ ; despite the restrictions on  $(l_N/d)$ associated with equation (28) these Stanton number results apply for  $(l_N/d) < 5$  for the lowest Reynolds number, 3250, for which results are quoted.

Results are also given in ref. [10], only for a Reynolds number of 4100, for a cylindrical nozzle,  $(l_N/d) = 1$ , surmounted by a 45° conical inlet of equal length, and for a sharp edged orifice formed by a 45° conical outlet of length equal to the inlet diameter. The hydrodynamic studies in ref. [8] indicated the flow to be laminar; the heat transfer performance is different for them, and those runs are identified, later, as IKL.

De Salve *et al.* [11] present results for the jet formed by a cylindrical nozzle,  $(l_N/d) \simeq 1.5$ , d = 1.9 mm, surmounted by a conical inlet; the authors appraised the jet to be relatively continuous. Three pressures were used in the test chamber, and for the lowest and highest Table 1 contains relations obtained by fairing through the rather scattered data points for  $Re < 10^4$ . For higher Reynolds numbers the Stanton number increased slightly as the Reynolds number increased. It is noted that the relations for the two pressures given in the table indicate a dependence on K greater than and opposite to that indicated by the formulation for the results of Sklover and Rodivilin.

Mills *et al.* [3] gave results for evaporation, the jet being formed by a long cylindrical nozzle with d = 4 mm. The results were presented graphically and the relation in Table 1 is a fairing through the results for which the Reynolds number ranged from  $10^4$  to  $2.3 \times 10^4$ .

The accuracy of the various results is hard to appraise, for Zakharov the results scatter substantially, but for the 3 and 5 mm nozzles  $\Delta St/St$ , which is  $\Delta \log T_R/\log T_R$  is about 0.10; for the 7.05 mm nozzle, it is about 0.22. There are so few results given by Zinger that no appraisal can be made. Table 1 contains estimates for the other results, based on departure of data points from the correlation indicated. Only for Isachenko *et al.* [5], Sklover and Rodivilin [9] and Iciek [10] are there enough data points to make the estimate reliable.

In view of the evident disparity between these various experimental results, it is important to note that, in so far as it is possible to determine from the description of the experimental systems, adequate venting for non-condensable gases was provided. Their presence in the vapor region would diminish the heating of the water and so reduce the Stanton number and, particularly, the venting appeared to be adequate in systems in which low Stanton numbers were obtained.

#### THE EVALUATION OF E

Table 2 contains the evaluation of E from the results for  $T_R$  as these are given graphically in ref. [1] for the Zakharov and for the Zinger experiments. The three jet velocities in column 1 of the table cover the experimental range, for the nozzle diameter and the jet length of columns 2 and 3. These, with the system pressure, p, and the initial jet temperature, are the basis of the parameters of columns 4–6, evaluated for the initial temperature. Column 7 is the temperature ratio,  $T_R$ ; this gives  $\xi$  from equation (16), and  $Ex^*F$ , and also F, are obtained from equation (18). These determine E, column 11. Column 12 is the estimate of the error in E from equation (24). For the Zakharov results the values of E for the two smaller diameters

Author	$h/\rho c_p u_0$	P (MPa)	<i>T</i> <sub>0</sub> (°C)	$\Delta St/S$
Isachenko et al. [5]	$C(d/l)^n \exp(0.135We')$	0.147-0.157	19-89	0.15
$a = 2.18 \text{ mm}$ $l_N/d = 46$	for $(l/d) < 95$ , $C = 0.0129$ , $n = 0.54$ (l/d) > 95, $C = 0.00375$ , $n = 0.27$			
	$We' = ( ho_{ m v} du_0^2/\sigma) = ( ho_{ m v}/ ho_{ m L})We$			
Sklover and Rodivilin [9] 3 < d < 20  mm	$0.02 \left(\frac{u_0 d}{v}\right)^{0.2} \left(\frac{\alpha}{v}\right)^{0.57}_{\rm L} K^{0.1} \left(\frac{d}{l}\right)^{0.75}$	0.015-0.098	29-38	0.15
l/d = ? small	$K = h_{\rm fg} / [C_p (T_{\rm s} - T_0)]$			
Iciek [10] 3 < d < 5  mm	$8.63 \times 10^{-3} \left(\frac{d}{l}\right)^{0.28} \left(\frac{u_0^2}{gd}\right)^{-0.10};  \frac{l}{d} < \left(\frac{l}{d}\right)_{\rm B}$	0.1013	24 and 40	0.03
$1 < (l_{\rm N}/d) < 9$	$8.75 \times 10^{-3} \left(\frac{d}{l}\right)^{0.22} \left(\frac{u_0^2}{gd}\right)^{-0.18};  \frac{l}{d} > \left(\frac{l}{d}\right)_{\rm B}$	0.1013	24 and 40	0.03
	for $(l/d)_{\rm B} = 11.5(\rho du_0^2/\sigma)^{0.31}$			
De Salve <i>et al.</i> [11] $d = 1.9 \text{ mm}$	$8.15 \times 10^{-3} \left(\frac{u_0 d}{v}\right)^{-0.16}$	0.183	36	0.07
	$10.60 \times 10^{-3} \left( \frac{u_0 d}{v} \right)^{-0.16}$	0.379		
	for $(l/d) = 163$ only			
Mills <i>et al.</i> [3] d = 4  mm long tube (evaporation)	$1.04\left(\frac{u_0d}{v}\right)^{-0.38}\left(\frac{\alpha}{v}\right)^{0.5}\left(\frac{d}{l}\right)^{0.5}$	0.001	18	0.03

Table 1. Summary of correlations for various experimental results

confirm the recommendation of  $E = 5 \times 10^{-4}$  that is made in ref. [1].

Table 2 also contains results from the numerical evaluation of equations (1)-(3) as that was made for a value of E typical of those found in column 11. This calculation was first carried out assuming no condensation,  $m_{\rm c} = 0$ , so that there would be no shear at the exterior of the jet. The initial incentive for this calculation was the uncertainty about equation (11) as noted in ref. [4] but by the development preceding equation (11) this uncertainty was largely unfounded. These numerical results, column 14, should check the analytical results, as given by column 13 for the assumed value of E. They do not check very well, possibly because of the truncation error in the numerical results, the worst correspondence being for the 3 mm nozzle at the lower velocities. The calculation was made also for a condensation flux as in equations (20) and (21). With this mass addition,  $T_{\rm m}$  is reduced and  $(T_s - T_0)/(T_s - T_m)$  should be smaller than without condensation. The numerical results, column 15, reveal such a trend for the results for d = 3 mm, though for d = 5.07 mm the results in columns 14 and 15 are essentially the same.

Table 3 contains the evaluation of E for the other experimental results. Two Reynolds numbers were chosen to represent the range of the experimental results; these or equivalently,  $(u_0^2/gd)$  for Iciek, were used to evaluate the Stanton number from Table 1 for values of l/d typical of the experimental range. Then,

 $T_{\rm R}$ , column 7, was obtained from the Stanton number. However, for Mills *et al.*, De Salve *et al.*, and for the results, IKL, for the non-cylindrical nozzles used by Iciek for Re = 4100, the Stanton numbers or of  $T_{\rm R}$  were obtained from the graphical representations of those results. The values of *E* are given in column 11. Except for those for Sklover and Rodivilin and for De Salve *et al.*, they are lower than those contained in Table 2.

Figure 1 summarizes the results for E by representation of most of the values from Tables 2 and 3 as a function of l/d, the parameters being the particular experiment, with two Reynolds numbers for each. The nozzles were apparently cylindrical, but of various lengths for all cases except Iciek, IKL. Except for that case and that of Zakharov, the value of  $E \times 10^4$ is between 1 and 2 for  $0.33 < Re \times 10^{-4} < 2.3$ , but the trend with Reynolds number differs for Mills et al. and Iciek, IK. Zakharov gives values of E that are much higher, as does De Salve et al. The De Salve et al. nozzle was not completely cylindrical and the values of E, not shown on the figure, are of the order of  $4 \times 10^{-4}$ . In view of this situation for the lower part of the Reynolds number range, there is little prospect of explanation for the higher Reynolds numbers, for which the values of  $E \times 10^4$  are as high as 50. Mills et al. [3] faced a similar dilemma in trying to rationalize the results for the Stanton number for these same experiments except for Iciek and for De Salve et al., not then available. Mills et al. considered that the

			IaDI	le 2. Eval	uation of E	for the Zak	harov an	d Chernay	/a (given	ın ret. [1	) and Zi	nger [2] ex	periment	al results				
		1	2	ŝ	4	5	9	7	~	6	10	11	12	13	14	15		
		$(m s^{-1})$	d (mm)	$\frac{1}{d}$	$\frac{u_0d}{v}$	$\frac{\alpha}{u_0r_0}\frac{l}{r_0}$	$\frac{2gl}{u_0^2}$	$\frac{T_{\rm s}\!-\!T_{\rm 0}}{T_{\rm s}\!-\!T_{\rm m}}$	<i>1</i> ,16	$E^{x*F}$	F	$E  imes 10^4$	$\frac{\Delta E}{E}$	$(T_{\rm s}-2)$ equation (16)	$\frac{T_0}{m_c} = 0$	$T_{\rm m}$ ) num. $m_{\rm c} \neq 0$		
Zakharov and	ZI	0.75	с Э	150	2250	0.038	15.7	8.0	0.296	0.258	1.67	5.1	0.14	7.67	7.35	7.09	for $E = 0.000$	)5
Chernaya [1]	Z	0.90			2700	0.032	10.9	6.6	0.260	0.228	1.55	4.9		6.67	6.41	6.25		
$T_0 = 20^{\circ} \text{C}$	Z3	1.40	:	:	4200	0.020	4.5	5.0	0.214	0.194	1.32	4.9	0.14	5.10	4.98	4.90		
p = 0.1013	Z4	0.70	5.07	88.7	3549	0.014	18.0	4.0	0.175	0.161	1.72	5.2		3.78	3.69	3.69		
	Z5	1.00	"	:	5070	0.010	8.8	3.5	0.152	0.142	1.48	5.4		3.27	3.21	3.22		
	Z6	1.50	:	:	7605	0.007	3.9	2.9	0.120	0.113	1.29	4.9		2.87	2.85	2.87		
	Z	0.50	7.05	42.6	3525	0.007	23.5	3.1	0.132	0.125	1.82	8.1	0.34	3.00		2.93	for $E = 0.00$	770
	Z8	0.60		:	4230	0.006	16.3	2.8	0.114	0.108	1.68	7.6		2.84		2.77		
	6Z	0.90	:	:	6345	0.004	7.3	2.5	0.094	060.0	1.43	7.4	0.37	2.57		2.49		
Zinger [2]	ZNI	10	10	80	$7.7 \times 10^{4}$	0.00044	0.16	9.3	0.322	0.322	1.02	20		9.50		9.44	for $E = 0.00$	20
$T_0 = 10^{\circ} \text{C}$	ZN2	25	10	80	$19.2 \times 10^{4}$	0.00018	0.03	145	0.797	0.797	1.00	50		145		137	for $E = 0.00$	20
p = 0.1013	ZN3	10	15	53.3	$11.5 \times 10^{4}$	0.00020	0.16	7.7	0.289	0.289	1.02	27		7.69		7.82	for $E = 0.00$	266
	ZN4	20	15	53.3	$23.1 \times 10^{4}$	0.00010	0.04	27	0.506	0.506	1.00	48		24.2		24.1	for $E = 0.00$	<del>1</del> 6



diameter and length of the nozzle might influence the results, but subsequently the experimental results of Iciek [10] showed that, as noted in connection with Table 1, the ratio  $(l_N/d)$  does not influence the Stanton number relation for cylindrical nozzles, at least for the relatively low Reynolds numbers of those experiments. Other factors must be considered, such as the amount of condensation and the length of the continuous portion.

Table 4 contains the relative condensation,  $m_c/m_0$ , evaluated by equation (22) from the value of  $T_R$  and the system pressure and initial jet temperature given in Tables 2 and 3, for some of the entries in those tables. The condensation rate is in most cases a relatively small fraction of the initial flow rate.

Table 4 also contains the jet break-up length,  $(l/d)_{\rm B}$ , as evaluated from equation (28), from the value of  $\sqrt{We}$  that is also contained in the table. Only for the Iciek, IKL, results for the short non-cylindrical nozzles were equations (25) and (27) used instead, and this was done also for the De Salve et al. run DS6 because of the low initial  $\sqrt{We}$  for that run. On the basis of this appraisal a continuous jet exists in the region  $0 < (l/d) < (l/d)_{\rm B}$ . Then disintegration occurs, and there is a drop flow for  $(l/d) > (l/d)_{\rm B}$ . The values of  $(l/d)_B$  obtained from equation (28) can, however, be viewed with confidence only to a limit of  $\sqrt{We}$  of about 40, the limit of the experiments that gave the results described by equation (28). This restriction makes questionable the values given in Table 4 for the experiments of Zinger, Sklover and Rodivilin and of Isachenko et al. for the high Reynolds number of  $3.2 \times 10^4$ , with  $\sqrt{We} = 82$ .

For the remaining results a continuous jet,  $(l/d) < (l/d)_B$ , is presumed to have existed, only for the results of Zakharov for the 7 mm diameter nozzle.

			5	6	7	8	9	10	11
		1	x l	2gl	$T_{\rm s} - T_{\rm 0}$	č	$E_{\lambda} * F$	F	$E \times 10^4$
		d	$u_0 r_0 r_0$	$u_0^2$	$T_s - T_m$	-			*** / 1 * 0
Isachenko et al. [5]					$(u_0 d/v) =$	$1.6 \times 10^{4}$			
$T_0 = 20^{\circ} \mathrm{C}$	11	30	0.0011	0.0237	1.32	0.0130	0.0119	1.0	1.99
d = 2.18  mm	12	60	0.0022	0.0473	1.47	0.0235	0.0213	1.0	1.78
	13	95	0.0034	0.0749	1.61	0.0341	0.0307	1.0	1.61
	I4	160	0.0058	0.1262	2.00	0.0631	0.0573	1.01	1.77
	15	200	0.0072	0.1577	2.27	0.0821	0.0749	1.02	1.83
					$(u_0 d/v) =$	$3.2 \times 10^{4}$			
	I6	30	0.0005	0.0059	1.52	0.027	0.027	1.0	4.41
	17	60	0.0011	0.0118	1.78	0.047	0.046	1.0	3.83
	18	95	0.0017	0.0187	2.05	0.066	0.064	1.0	3.38
	19	160	0.0029	0.0315	2.84	0.117	0.114	1.0	3.54
	110	200	0.0036	0.0394	3.42	0.149	0.145	1.0	3.61
Mills et al. [3]					$(u_0 d/v) =$	$1.0 \times 10^{4}$			
$T_0 = 18^{\circ}C$	M1	24	0.0013	0.271	1.28	0.0105	0.0092	1.03	1.86
p = 0.001  MPa	M2	33	0.0018	0.373	1.32	0.0130	0.0109	1.04	1.63
d = 4  mm	M3	43	0.0023	0.486	1.39	0.0175	0.0148	1.05	1.67
					$(u_{i}d_{i}v) =$	$2.3 \times 10^{4}$			
	M4	24	0.0006	0.051	1 19	0.0060	0.0054	10	1.12
	M5	23	0.0008	0.071	1.23	0.0035	0.0067	1.0	1.01
	MG	43	0.0000	0.071	1 27	0.0075	0.0085	1.01	0.09
	MO	43	0.0010	0.092	1	0.0075	0.0085	1.01	0.20
Sklover and Rodivilin [9]				$u_0 d/v) = 4.$	$15 \times 10^4$ ; d	= 3 mm; <i>i</i>	$u_0 = 10 \text{ m s}^{-1}$	• 1	
$T_0 = 35^{\circ}\mathrm{C}$	S1	20	$4.0 \times 10^{-4}$	0.012	2.03	0.065	0.065	1.0	16.2
p = 0.098  MPa	S2	80	0.0016	0.047	2.73	0.110	0.108	1.0	6.72
	S3	120	0.0024	0.071	3.05	0.129	0.127	1.0	5.23
	S4	200	0.0040	0.118	3.54	0.155	0.151	1.01	3.71
			(1	$u_0 d/v) = 27$	$.7 \times 10^4$ ; d =	= 20 mm;	$u_0 = 10 \text{ m s}$	- 1	
	S5	20	$6.0 \times 10^{-5}$	0.079	2.80	0.114	0.114	1.0	28.2
	S6	80	$2.4 \times 10^{-4}$	0.31	4.32	0.189	0.189	1.04	11.4
	<b>S</b> 7	120	$3.6 \times 10^{-4}$	0.47	5.05	0.216	0.216	1.05	8.5
	<b>S</b> 8	200	$6.0 \times 10^{-4}$	0.78	6.30	0.254	0.254	1.08	5.8
Iciek [10]				(u,d v)	$= 3290 \cdot d =$	= 4 mm · 7	$C_{\rm e} = 24^{\circ} C$		
leiek [10]	IKI	20	0.0039	2 78	1 26	0.0090	0.0051	1 23	1.04
	IK2	40	0.0078	5 56	1.45	0.0225	0.0147	1 37	1.34
	IK 3	80	0.0156	11.12	1.90	0.0550	0.0394	1.55	1.58
	116.3	120	0.0234	16.68	2 41	0.0920	0.0687	1.69	1.69
	115.7	120	0.0254	d(w) = 007	17  for  l/d < 1	$40 \cdot d = 5$	mm: T - 40	н <b>с</b>	
			(u	d(v) = 967	13 for $1/d > 13$	40, d = 5 $40 \cdot d = 5$	mm, $T = 40mm$ , $T = 40$	r C	
	IK 5	20	0.0019	1.14	1.23	0.0072	0.0053	1.11	1.19
	IK6	40	0.0037	2.29	1.40	0.0190	0.0153	1.20	1.59
	IK7	80	0.0077	4.87	1.76	0.0455	0.0378	1.34	1.76
	IK8	120	0.0115	7.30	2.17	0.0758	0.0643	1.43	1.87
Iciek [10]				$(u_0 d/v)$	= 4100; d =	= 5 mm; 7	$\Gamma_0 = 24^{\circ}C$		
leter [10]	IKLI	20	0.0031	3.50	1.16	0.00347	0.00037	1.27	0.072
	IKL2	40	0.0062	6.99	1.26	0.0090	0.0028	1.42	0.25
	IKL3	80	0.0125	13.99	1.60	0.0330	0.0205	1.63	0.79
	IKI 4	120	0.0187	20.98	2.17	0.0755	0.0568	1.78	1.33
	IKL5	160	0.0250	27.97	2.86	0.1178	0.0928	1.89	1.53
				11.1 11	62 + n = 0.19	22 MDo - 7			
De Salve et al. [11]	DEI	5400	0.0257	150	$\frac{00}{40}$	0.176	0.150	1 14	4.0
	D31 D\$2	0040	0.0257	0.53	3 77	0.175	0.150	1.06	43
	D32 D92	11 500	0.0104	0.33	3.77	0.138	0.125	1.00	37
	D33	16 500	0.0121	0.16	3.12	0.132	0.124	1.02	37
	D34 D85	27300	0.0051	0.10	3.45	0.150	0.145	1.00	4.4
	100	21500	0.0051	lld = 1	$63 \cdot n = 0.3$	79 MPa · 7	$r_{0} = 36^{\circ}C$		
	Dee	1150	0.121	u/u = 10 33.04	(3, p = 0.3) 8 33	0 303	0.182	1.97	2.8
	D30	1130	0.121	20.04	7 70	0.289	0.253	1.24	6.3
	DS/	5400	0.030	1.50	5 55	0.232	0.207	1 14	5.5
	D20	0030	0.020	0.54	5.00	0.214	0.199	1.06	5.8
	D37	12700	0.0109	0.27	4 44	0 194	0.183	1.03	5.4
	DS11	24 600	0.0057	0.07	5.00	0.214	0.208	1.01	6.3
		27 UUU	0.0007	0.07	2.00	··-· ·			

Table 3. Evaluation of E for the other experimental results

	7	$\frac{m_{\rm c}}{m}$		(1)
Case	$\frac{1}{d}$	(equation (22))	√We	$\left(\frac{1}{d}\right)_{B}$
Z1	150	0.127	4.9	31
Z3	150	0.115	9.1	45
Z4	88.7	0.107	5.9	34
Z6	88.7	0.092	12.7	56
Z7	42.6	0.096	5.0	31
Z9	42.6	0.084	9.0	45
ZN1	80	0.13	115	217
ZN2	80	0.15	288	385
ZN3	53	0.13	141	286
ZN4	53	0.14	282	450
I1	30	0.032	41	115
15	200	0.079		
16	30	0.046	82	176
110	200	0,101		
M1	24	-0.004	19	71
M3	43	-0.005		
M4	24	-0.003	47	125
M6	43	-0.004		
<b>S</b> 1	20	0.057	68	157
S4	200	0.083		
S5	20	0.073	171	278
S8	200	0.100		
IK1	20	0.026	5.6	33
IK4	120	0.078		
IK5	20	0.024	11.1	51
IK8	120	0.072		
IKLI	20	0.017	6.2	152†
IKL4	120	0.071		ŧ
DSI	163	0.111	4.5	49
DS2	163	0.105	53	135
DS6	163	0.177	2.2	52†
DS10	163	0.152	28.7	92

Table 4. Evaluation of relative condensation, Weber number and continuous jet length for the experimental results

† From equations (25) and (27).

for which  $E \times 10^4$  has been found to be like 7; for the low Reynolds number results of Isachenko *et al.*, for (l/d) < 115 for which  $E \times 10^4$  varies from 2 to 1.6; all of Mills *et al.*,  $1 < E \times 10^4 < 1.9$ ; Iciek for (l/d) < 33and 51, giving  $E \times 10^4$  values of 1.04 and 1.19. The results of Zakharov are distinguished by their high value of *E*. The De Salve *et al.* results are not included in this group because, by equation (28),  $(l/d) > (l/d)_B$ , though admittedly, De Salve *et al.* [11] indicated that, by observation, their jet was relatively continuous, though the flow rate and jet length for which this was observed was not mentioned.

As (l/d) is increased beyond  $(l/d)_B$  for a given operating condition, the value of *E* should reflect, increasingly, the effect of the transfer in the region of dispersed flow. For Zakharov, for  $Re \sim 4000$ , *E* is about  $5 \times 10^{-4}$  for the two nozzles of smaller diameter, even for the smallest diameter, for which (l/d) = 150and the region of dispersed flow would dominate. For Isachenko *et al.*, for 115 < (l/d) < 200, *E* increases slightly, but the average value there differs little from the average value for the continuous region. For Iciek,  $(l/d)_{\rm B} = 33$ , the value of *E* increases from about 1.3 to 1.7 in the dispersed region, and for the higher Reynolds number, from about 1.7 to 1.9.

There is no consistent trend for E in any of these results. Even for the presumably continuous jet, Zakharov indicates essentially no dependence with Reynolds number, Mills *et al.* indicate lower values for the higher Reynolds number, and Iciek gives a slightly higher value for the higher Reynolds number. Moreover, as noted already, the values of E are much higher for the Zakharov results. It is apparent, from Table 4, that for a continuous jet the relative condensation values are in the sequence Zakharov, Isachenko *et al.*, Iciek and Mills *et al.* (the evaporation case), but the values of E for all of the latter are about the same and no more specific effect for the relative condensation is apparent.

Finally it is appropriate to comment on the source and magnitude of the diffusivity and of the factor Ethat is used in its definition. This is in part due to the conditions at the nozzle outlet and for a long enough cylindrical nozzle,  $(l_N/d)$  greater than about 10, a turbulent flow should be, or nearly be, fully developed. For such a fully developed flow the eddy diffusivity is relatively uniform for all radii except in the region of the wall, with the value

$$\frac{\varepsilon_{\rm M}}{v} = 0.07 \frac{\bar{u}r_0}{v} \sqrt{\left(\frac{\tau_0}{\rho \bar{u}^2}\right)}.$$
 (29)

With  $(\tau_0/\rho \bar{u}^2)$ , which is half the friction coefficient, evaluated by a power law expression for a smooth pipe

$$\frac{\varepsilon_{\rm M}}{v} = \frac{\bar{u}r_0}{v} \left[ 0.014 \left( \frac{v}{\bar{u}d} \right)^{1/8} \right].$$

If this diffusivity, for the central part of the exit flow, is taken to be that of the entire jet, which view neglects the lower diffusivity of the wall region that at least initially remains near the jet surface, and if the diffusivity for heat is taken equal to that for momentum, then

$$E = 0.014 \left(\frac{v}{\bar{u}d}\right)^{1/8}.$$
 (30)

Dissipation will reduce the diffusivity as the distance from the nozzle exit is increased unless there is an input from a shear at the jet surface. With the velocity of the jet different from that of the surrounding vapor this shear exists. With condensation it is relatively large and it is dominated by the momentum transfer due to the inward mass flux; with evaporation the shear is diminished because of the outward flux (blowing). This shear will reduce the rate at which the diffusivity decreases, and if it is great enough, might increase it.

The shear at the jet surface cannot be evaluated well, particularly for the irregular wavy surface of the jet. Condensation alone was considered in equation (21) and if that is done

$$\frac{\tau}{\rho u^2} = \frac{-\rho uv}{\rho u^2} = \frac{m'}{2\pi r_1 \rho u} = \frac{r_1 m'}{2\pi r_0^2 \rho u_0}.$$

If condensation is neglected for the evaluation of the denominator, that is  $2m_0$ . If in the numerator  $r_1$  is taken as  $r_0$  and m', which decreases from large (analytically infinite) initial values, is approximated as  $m_c/l$ , then

$$\frac{\tau}{\rho u^2} = \frac{1}{4} \frac{d}{l} \frac{m_c}{m_0}$$

If, rather arbitrarily equation (29) is used then

$$E = 0.035 \sqrt{\left(\frac{d}{l} \frac{m_c}{m_0}\right)}.$$
 (31)

As an example, for  $Re = 10^4$ , equation (30) gives  $44 \times 10^{-4}$ . This is much higher than any of the values of Fig. 1 for a nozzle Reynolds number of this magnitude. The evaluation of equation (31) requires the choice of an experiment so that equation (22), or equivalently, Table 4, can be used. For example, the Iciek results for this Reynolds number give, for (l/d) = 20,  $E \times 10^4 = 12$  and for (l/d) = 120,  $E \times 10^4 = 8.5$ . The values in Table 3 are 1.19 and 1.87. These comparisons are unfavorable but they do give some insight as to the diffusivity in the jet.

#### SUMMARY

This consideration, of available results for heat transfer to liquid jets falling vertically in a region of its vapor, in terms of the Kutateladze theory produced values of E, the factor in the diffusivity specification, as diverse as the values of the Stanton number that expressed the heat transfer performance of the jets. There is uncertainty about the length of the continuous region of the jets in the various exper-

imental systems but even a conservative estimate for this region, only to which the Kutateladze theory can logically be applied, still contains a range of values of E which cannot be rationalized. The dilemma of Mills *et al.* [3] in viewing the wide range in the results for the Stanton numbers from the various experiments is, therefore, not resolved by the present consideration and the design problem for these jet systems remains unsolved, despite the large number of results from experiments on them. It is, in fact, difficult to suggest the nature of further experimental work by which the situation might be clarified. Certainly, however, the specification of the length of the continuous portion of the jet should be made more definitive.

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## CHAUFFAGE D'UN JET TURBULENT D'EAU SE DECHARGEANT VERTICALEMENT DANS UN ENVIRONNEMENT DE VAPEUR D'EAU

**Résumé**—Des résultats expérimentaux sur le chauffage de jets d'eau turbulents se déchargeant vers le bas dans un environnement de vapeur d'eau sont examinés à la lumière de la théorie de Kutateladze. Cette théorie définit une diffusivité turbulente pour la chaleur, proportionnelle au nombre de Reynolds local du jet,  $\varepsilon_{\rm H}/v = E(ur_1/v)$ , et le facteur E est évalué à partir des résultats expérimentaux. Le large domaine des valeurs de E ainsi obtenu est essentiellement inexplicable et la clarification du problème du chauffage du jet d'eau reste non résolu en dépit de l'effort expérimental important qui lui a été réservé.

#### DIE AUFHEIZUNG EINES TURBULENTEN WASSERSTRAHLS BEIM SENKRECHTEN EINSTRÖMEN IN EINEN DAMPFRAUM

**Zusammenfassung**—Es werden verfügbare experimentelle Ergebnisse über die Aufheizung eines turbulenten Wasserstrahls beim abwärts gerichteten Einströmen in einen Dampfraum gemäß der Kutateladze-Theorie dargestellt. Nach dieser Theorie wird eine scheinbare Temperaturleitfähigkeit  $e_{in}/v = E(ur_1/v)$  definiert, die proportional der örtlichen Reynolds-Zahl des Strahls ist. Der Faktor E wird aus den experimentellen Ergebnissen berechnet. Der so ermittelte weite Wertebereich des Faktors E bleibt im wesentlich unerklärbar. Die Berechnung der Aufheizung eines Wasserstrahls bleibt damit ungelöst, trotz der sehr umfangreichen experimentellen Untersuchungen über dieses Problem.

## НАГРЕВ ТУРБУЛЕНТНОЙ СТРУИ ВОДЫ, ИСТЕКАЮЩЕЙ ВЕРТИКАЛЬНО В СРЕДУ ВОДЯНОГО ПАРА

Аннотация — Имеющиеся экспериментальные результаты по нагреву турбулентных водяных струй, истекающих вниз в среду водяного пара, проанализированы с использованием теории Кутателадзе для такой системы. Эта теория определяет турбулентную теплопроводность, пропорциональную локальному числу Рейнольдса струи,  $e_{\rm H}/v = E(ur_1/v)$ , а коэффициент E оценивается по экспериментальным результатам. Большой диапазон изменения полученных таким образом значений E остается необъяснимым и, таким образом, задача по определению нагрева водяной струи остается нерешенной.